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# Wind speed dynamical model in a wind farm

Maryam Soleimanzadeh and Rafael Wisniewski

**Abstract**—This paper presents a model for wind speed in a wind farm. The basic purpose of the paper is to calculate approximately the wind speed in the vicinity of each wind turbine in a farm. In this regard the governing equations of flow will be solved for the whole wind farm. In ideal circumstances, the dynamic model for wind flow will be established. The state space variables are determined based on a fine mesh defined for the farm. The end goal of this method is to assist the development of a dynamical model of a wind farm that can be engaged for better wind farm control strategies.

## I. INTRODUCTION

Most of the wind farm models has been produced in the 1980's, but nowadays the Wind farms are larger and further research is required, to examine the performance of wind farm and wind models in more difficult conditions [1]. Presenting a proper wind speed model for a wind farm location is a complicated process[2]. In some researches wind speed model consists of two elements: a slowly varying mean wind speed of hourly average and a rapidly varying turbulence component, where mean wind speed is considered almost constant through the observation period. The component is modeled by a normal distribution with a null mean value and a standard deviation that is proportional to the current value of the mean wind speed [3], [4]. A common wind speed model for wind farms uses auto regressive time-series and model of moving average. In this method actual wind speed data that are collected hourly for a specific location, are used to construct a wind speed simulation model for a wind farm[5]. This model can express the correct probabilistic characteristics of wind flow for the wind farm [2]. In another approach to the problem of modeling of large wind farms wind turbines acted as distributed roughness elements, and they modified the ambient atmospheric flow. One of the main issues in wind farm modeling are the wake interaction and the velocity deficit and turbulence increase at the interaction locations[6]. The ambient basic flow, is to some extent affected by the wakes of the upstream turbines; therefore a good approach would be to consider the flow equations for the whole wind farm instead of its part. The engineering models that recently applied for calculating wake effects on neighboring wind turbines are mostly based on local momentum equations, neglecting the atmospheric interactions. There are some other models, which predict the efficiency of large wind farms by considering each turbine as a roughness element [7]. A third option is to apply computational fluid dynamics (CFD) schemes. There is a

considerable distance between CFD models and engineering analytical solutions, and a connection is needed between them in order to prepare more information for better design of the turbines and wind farms and development of more efficient control algorithms [1]. In this category, one version of the wind flow model in a farm was based on quasi-steady wake deficits computed in a loop involving a connection between a CFD actuator disc model of the wake interaction and an aeroelastic model. The flow problem was solved using a finite difference scheme, with initial conditions defined by the downstream wake deficit [8]. Using finite volume methods instead of finite difference is preferred; since, finite volume methods are basically a generalization of the finite difference method but they use the integration of the flow governing equations, instead of their differential form and this concludes in greater flexibility[9].

In this paper we have derived a dynamic state space model for the wind flow in the wind farm. The model is ready to use for control, estimation and dynamic simulation purposes. In order to validate the model, calculation results are compared with measurements and the absolute error has been shown and error sources are discussed. The end goal of this method is to assist the development of an analytical expression of an offshore wind farm wake, nevertheless the turbulent effect has not been considered.

## II. MODELING

### A. Overall

Most studies on wind farm modeling have made a division between the near-wake and far-wake regions; the near wake is defined as the region just behind the rotor; where the effect of the rotor is considerable [11]. The near-wake region is described as an area having a high turbulence generated by the blades, shear, and the reduction of tip vortices. The far wake is the region just behind the near wake. One of the objectives of modeling in the far-wake region based on [12] is to evaluate the effect of wind turbines on wind flow in farms [12]. Our approach focuses on this region. In far-wake region, neglecting the ambient shear flow, both of the velocity deficit and turbulence intensity can be assumed to be axis-symmetric. In this case the only characteristic of the wind turbines that entered as a parameter in these equations is the thrust coefficient of the turbine and also the summation of the rotor kinetic energy [6]. The flow equation for the whole wind farm, which is explained by Navier-Stokes equations, is considered to be axis-symmetric and gradients of the standard deviation, turbulent and viscous terms are neglected. In this case flow can be described with the two dimensional Reynolds equation in the thin shear layer approximation in

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Cartesian co-ordinates. This equation is combined with the continuity equation to form the overall differential equation system.

$$\frac{\partial}{\partial x}(\Gamma \frac{\partial U}{\partial x}) + \frac{\partial}{\partial y}(\Gamma \frac{\partial U}{\partial y}) + S = \rho \frac{\partial U}{\partial t} \quad (1)$$

Where  $U$  is the mean wind speed,  $\Gamma$  is the diffusion coefficient,  $\rho$  is the air density, and  $S$  is the source term (that is considered as pressure gradient). We seek the solution  $U(x, y, t)$  subject to appropriate conditions on the boundary. Wind farm is divided into non-overlapping control volumes by the lines that define the boundaries of the individual control volumes, in here the control volumes are squares, but this is not a requirement. Fig. 1 depicts an example configuration of wind turbines placed in a row, and Fig. 2 shows typical meshes for this problem and some indexes for each control volume (CV). The finite volume method is used to transform (1) to a system of discrete equations for the nodal values of  $U$ . First, (1) is integrated over the typical control volume depicted in Fig. 3. This reduces each equation to one involving only first derivatives in space. Then these first derivatives are replaced with central difference approximations [13].

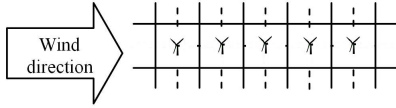


Fig. 1: An example configuration of wind turbines

The control volume integration is the most important step of the finite volume method, and yields the equation (2). It

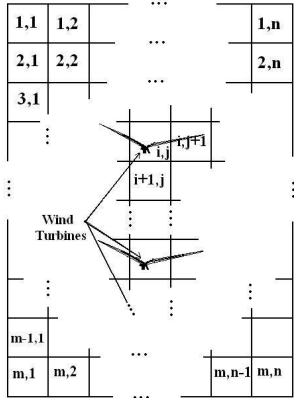


Fig. 2: Typical computational grid with indexes shown for each CV

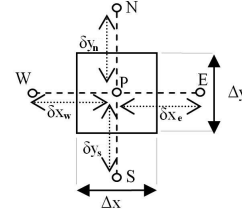


Fig. 3: A control volume

should be noticed that in this step the integration is just over the CV, and not over the time interval.

$$\int_{cv} \left[ \frac{\partial}{\partial x}(\Gamma \frac{\partial U}{\partial x}) + \frac{\partial}{\partial y}(\Gamma \frac{\partial U}{\partial y}) \right] dv + \int_{cv} S dv = \frac{\partial}{\partial t} \left( \int_{cv} \rho U dv \right) \quad (2)$$

Central difference approximation for each term of the equation based on Fig. 3 would be as following [13].

$$\int_{cv} \frac{\partial}{\partial x}(\Gamma \frac{\partial U}{\partial x}) dv \approx \left[ \Gamma \frac{U_E - U_P}{\delta x_e} - \Gamma \frac{U_P - U_W}{\delta x_w} \right] \Delta y \quad (3)$$

$$\int_{cv} \frac{\partial}{\partial y}(\Gamma \frac{\partial U}{\partial y}) dv \approx \left[ \Gamma \frac{U_N - U_P}{\delta y_n} - \Gamma \frac{U_P - U_S}{\delta y_s} \right] \Delta x \quad (4)$$

$$\int_{cv} S dv \approx S_P \Delta x \Delta y \quad (5)$$

$$\frac{\partial}{\partial t} \int_{cv} \rho U dv \approx \rho \frac{\partial U_P}{\partial t} \Delta x \Delta y \quad (6)$$

After substitution equations (3)-(6) into (2) and rearrangement, we have:

$$\rho \frac{\partial U_{P_i}}{\partial t} + a_P U_{P_i} - a_E U_{E_i} - a_W U_{W_i} - a_N U_{N_i} - a_S U_{S_i} = b_i \quad (7)$$

in which the coefficients  $a_j$ ,  $j = P, E, W, N, S$  are defined in table I.

It can be shown that similar equations can be obtained for other cells as well. Wind turbines are modeled as momentum absorbers by means of their thrust coefficient (actuator disk approach). An algorithm for velocity updating is applied, which maintains the compatibility of the velocity and thrust coefficient based on the following assumptions. With respect to wind speed in a time interval an average constant thrust coefficient,  $C_T$ , can be obtained using thrust curve for

Coefficients	Description
$a_E$	$\frac{-\Gamma}{\Delta x \delta x e}$
$a_W$	$\frac{-\Gamma}{\Delta x \delta x w}$
$a_N$	$\frac{-\Gamma}{\Delta y \delta y n}$
$a_S$	$\frac{-\Gamma}{\Delta y \delta y s}$
$a_P$	$-(a_E + a_W + a_N + a_S)$
$b$	$S_P$

**TABLE I:** coefficient of the equations

each wind turbine. The source term is proportional to wind pressure gradient that should be updated in computation loop for each turbine in the farm [14]:

$$a_i = -\frac{1}{2}(1 + \sqrt{1 - C_{T_i}}) \quad (8)$$

$$U_{after_i} = (1 - 2a_i)U_{before_i} = (f_i)U_{before_i} \quad (9)$$

$$\Delta P_i = \frac{1}{2}\rho(4a_i(1 - a_i))U_{before_i} \quad (10)$$

In which  $a_i$  is defined as the axial flow interference factor for  $i^{th}$  wind turbine [14],  $i = 1, \dots, 5$  is number of wind turbine.  $\Delta P_i$  is the pressure difference before and after the  $i^{th}$  turbine. In the next section it will be shown how above mentioned equations will appear in the state space model of the system.

### B. State Space Form

The state space form of the mentioned equations are as following. The state vector  $X$  is defined as (11) based on the mesh shown Fig. 2.

$$X = \begin{bmatrix} U_{P_{1,1}} \\ U_{P_{1,2}} \\ \vdots \\ U_{P_{1,n}} \\ U_{P_{2,1}} \\ U_{P_{2,2}} \\ \vdots \\ U_{P_{2,n}} \\ \vdots \\ U_{P_{m,1}} \\ U_{P_{m,2}} \\ \vdots \\ U_{P_{m,n}} \end{bmatrix} \quad (11)$$

Rewriting the dynamic equations of the wind farm with respect to new variables, yields to:

$$\dot{x}_1 = \frac{-1}{\rho} (a_P x_1 - [b_{1,1} + a_E U_{E_{1,1}} + a_W U_{W_{1,1}} + a_N U_{N_{1,1}} + a_S x_{S_{1,1}}]) \quad (12)$$

$$\dot{x}_2 = \frac{-1}{\rho} (a_P x_2 - a_W x_1 - [b_{1,2} + a_E U_{E_{1,2}} + a_N U_{N_{1,2}} + a_S x_{S_{1,2}}]) \quad (13)$$

$\vdots$

$$\dot{x}_n = \frac{-1}{\rho} (a_P x_n - a_W x_{n-1} - [b_{1,n} + a_E U_{E_{1,n}} + a_N U_{N_{1,n}} + a_S x_{S_{1,n}}]) \quad (14)$$

$$\dot{x}_{n+1} = \frac{-1}{\rho} (a_P x_{n+1} - a_N x_1 - [b_{2,1} + a_E U_{E_{2,1}} + a_N U_{N_{2,1}} + a_S x_{S_{2,1}}]) \quad (15)$$

$$\dot{x}_{n+2} = \frac{-1}{\rho} (a_P x_{n+2} - a_W x_{n+1} - a_N x_2 - [b_{2,1} + a_E U_{E_{2,2}} + a_S x_{S_{2,2}}]) \quad (16)$$

$\vdots$

$$\dot{x}_{2n} = \frac{-1}{\rho} (a_P x_{2n} - a_W x_{2n-1} - a_N x_n - [b_{2,n} + a_E U_{E_{2,n}} + a_S x_{S_{2,n}}]) \quad (17)$$

$\vdots$

$\vdots$

$$\dot{x}_l = \frac{-1}{\rho} (a_P F_{trb} x_l - a_W x_{l-1} - a_N x_{l-n} - [b_{i,j} + a_E U_{E_{i,j}} + a_S x_{S_{i,j}}]) \quad (18)$$

$$\dot{x}_{l+1} = \frac{-1}{\rho} (a_P F_{trb} x_{l+1} - a_W x_l - a_N x_{l+1-n} - [b_{i,j+1} + a_E U_{E_{i,j+1}} + a_S x_{S_{i,j+1}}]) \quad (19)$$

$\vdots$

$$\dot{x}_{l+n} = \frac{-1}{\rho} (a_P x_{l+n} - a_W x_{l+n-1} - a_N F_{trb} x_l - [b_{i+1,j} + a_E U_{E_{i+1,j}} + a_S x_{S_{i+1,j}}]) \quad (20)$$

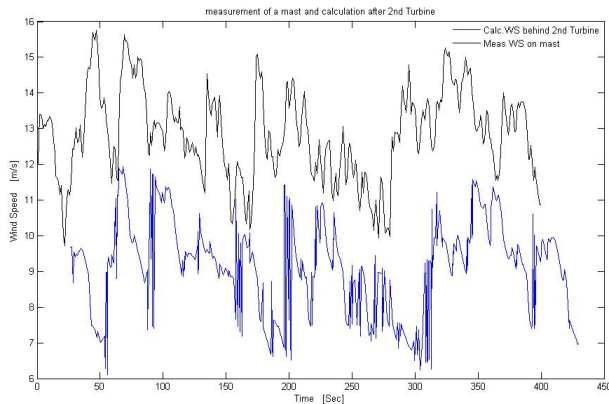
$\vdots$

$$\dot{x}_{mn} = \frac{-1}{\rho} (a_P x_{mn} - a_W x_{mn-1} - a_N x_{mn-n} - [b_{m,n} + a_E U_{E_{m,n}} + a_S x_{S_{m,n}}]) \quad (21)$$

In which  $F_{trb}$ ,  $trb = 1, \dots, 5$  are as in equation (9) and are proportional to the control input.  $b_{i,j}$ , which are the source terms and obtain from equation (10) are also related to the control input. So above mentioned equations are the dynamic equations of the wind farm and can be easily written in the form of  $\dot{X} = A(t, u)X + B(t, u)$ , which is the overall form of the dynamical model.

### III. SIMULATION AND VALIDATION

Validation inspects if the computational or conceptual models as executed into the CFD code, and computational simulation concur with real world informations. The method is based on error and uncertainty identification via comparison of simulations with measurement data [15]. Simulation is performed for the situation shown in Fig. 1 in MATLAB environment, where the measurements of a meteorological mast installed outside the farm are boundary conditions for the wind farm in each time. The black plot depicted in Fig. 4 shows wind speed in any second before the wind farm, and the other one is the calculated wind speed after the second turbine, for instance. As it can be seen in this picture, the shape of the calculated wind speed behind the second turbine is almost the same as measured wind speed; though behind next turbines the variations in the shape are larger. Wind speed measurements and thrust coefficients of turbines are based on reference [16]. The time shift between the calculated wind speed and measurement data is caused by the distance between turbines (The distance between each wind turbine is about 300m).

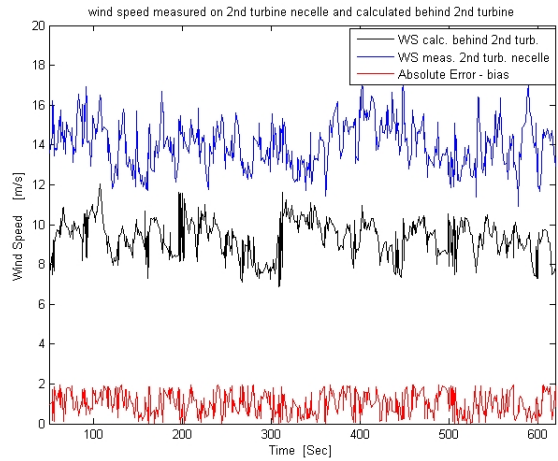


**Fig. 4:** wind speed measured on the mast before wind farm, and calculated wind speed behind the second turbine

In order to validate the calculations, they are compared with some measurements available by instruments mounted on each wind turbine nacelle. Some results are shown in Fig. 5 and 6; in each figure the red graph shows the absolute

error, after omitting the bias error. As it can be seen in most of the time the absolute error is less than 2m/s, that is still a considerable error. Some of the sources of uncertainties and errors are due to:

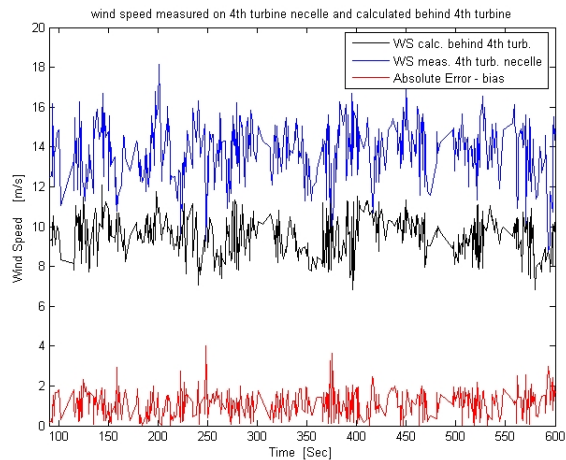
- 1) The measurements are referenced to nacelle of the turbine but calculations are supposed to be in the far wake region - there are no measurement available in that region. The model will be evaluated and adjusted with far wake region references in the near future.
- 2) Thrust curves and thrust coefficients are not accurate.
- 3) Turbulent flow model: turbulence characteristics of the wind flow is not considered in this work.
- 4) Selected mesh grid is one of the most important error sources. The selected mesh in here, to resolve the model, may not be the best selection, and using finer meshes the answer may be improved.



**Fig. 5:** Wind speed measured on the 2nd turbine nacelle and calculated behind the 2nd turbine, and the absolute error

### IV. CONCLUSIONS

In this paper a dynamical model is developed for wind speed in a wind farm. The model delivers an approximation of the wind speed in the vicinity of each wind turbine of a farm. In this regard, the wind farm is divided into non-overlapping cells and the flow equation is specified to each cell, such that the flow equation agrees on the boundaries of the cells. Spatial discretization for these equations is performed using finite volume techniques. Validation of the model is investigated by calculating the wind speed, where the effect of wind turbines is observed by means of their thrust coefficient. The scheme is based on finite volume discretization and can be used for better wind farm



**Fig. 6:** Wind speed measured on the 4th turbine nacelle and calculated behind the 4th turbine, and the absolute error

design and for more sophisticated control strategies and load calculations.

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